33.34 An $8in$ standard weight steel pipe (I.D. $7.98in$, O.D. $8.625in$) carries $500gpm$ of water initially at $200°F$. Insulation must be specified to limit the heat loss to $1°F$ per $500ft$ of length. The ambient temperature in the space around the pipe is $70°F$. Assume the inside and outside film coefficients on either side of the pipe are infinite and the thermal resistance of the pipe wall is negligible. The thermal conductivity of the insulation is $0.05\frac{Btu \cdot ft}{hr \cdot ft^2 \cdot °F}$. The film coefficient for the outside of the insulation is assumed to be infinite. What is the minimum required thickness for the insulation?

A. $\frac{1}{4}in$

B. $\frac{3}{8}in$

C. $\frac{1}{2}in$

D. $\frac{3}{4}in$

Start with the equation for Conduction Through a Cylindrical Wall (Heat Loss Through a Pipe).

$$\dot{Q} = \frac{2\pi kL\left(T_1 - T_2\right)}{ln\left(\frac{r_2}{r_1}\right)}$$

The thermal conductivity of the insulation, k, is known. The length, L, is given. The difference in temperature is from the water to the ambient air. The outer radius, r_2, is for the outside of the insulation, and the inner radius, r_1, is for the inside if the insulation i.e. the outside of the pipe. Assume all film coefficients are infinite, therefore there is no contact resistance or natural convection, nor does the pipe provide any insulating properties, as it is assumed to be a perfect conductor. The sole source of thermal resistance is the insulation.

Since the problem is asking for the thickness of the insulation, define t as the difference between the inner and outer radius of the insulation.

$$t = r_2 - r_1$$

$$r_2 = r_1 + t$$

The ratio of the outer and inner radii can then be expressed as below.

$$\frac{r_2}{r_1} = \frac{r_1 + t}{r_1} = 1 + \frac{t}{r_1}$$

Substitute into the original equation. Determine r_1 for use at the end.

$$\dot{Q} = \frac{2\pi kL\left(T_1 - T_2\right)}{ln\left(1 + \frac{t}{r_1}\right)}$$

$$r_1 = \frac{8.625in}{2} = 4.3125in$$

The allowable heat loss, $\dot{Q}$, escaping the pipe over a $500ft$ length is established with the sensible heating/cooling rule of thumb for water.

$$\dot{Q} = 500gpm\Delta T$$

$$\dot{Q} = 500\,(500)\,(1) = 250,000\frac{Btu}{hr}$$

Algebraically solve for the insulation thickness, t, substitute known values, and evaluate. Track units carefully. Note the ΔT is across the total resistance, from the water inside the pipe to the ambient air in the room.

$$ln\left(1+\frac{t}{r_1}\right) = \frac{2\pi kL\,(T_1 - T_2)}{\dot{Q}}$$

$$1+\frac{t}{r_1} = e^{\left(\frac{2\pi kL(T_1 - T_2)}{\dot{Q}}\right)}$$

$$t = \left[e^{\left(\frac{2\pi kL(T_1 - T_2)}{\dot{Q}}\right)} - 1\right]r_1$$

$$t = \left[e^{\left(\frac{2\pi\left(0.05\frac{Btu\cdot ft}{hr\cdot ft^2\cdot {}^\circ F}\right)(500ft)\left(200^\circ F - 70^\circ F\right)}{250,000\frac{Btu}{hr}}\right)} - 1\right](4.3125in) = 0.367in$$

Answer B